proportion of heat transfer by the quenching mechanism may also be explained by the increased presence of nucleate boiling.

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ON THE NONLINEARITY OF TWO-DIMENSIONAL HEAT TRANSFER IN A CONDUCTING AND RADIATING MEDIUM*

V. J. LUNARDINI† and Y. P. CHANG

Department of Mechanical Engineering, State University of New York at Buffalo, Buffalo, New York, U.S.A.

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NOMENCLATURE

h_s,	$3\epsilon_s/2(2 - \epsilon_s)$;
k,	thermal conductivity;
l,	$\varkappa L$, optical thickness of medium;
n,	inward normal at boundary;
N,	$k\varkappa/(4\sigma T_0^{*3})$;
p,	arccos (T_l/T_0);
$\vec{q}^*$,	heat flux;
$\vec{q}$,	$\vec{q}^*/\sigma T_0^{*4}$;
T^*,	temperature;
T,	T^*/T_0^*;
x^*, y^*,	physical coordinates;
x,	$\varkappa x^*$, $y = \varkappa y^*$, optical coordinates;
∇^2,	$\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$;
ϵ,	emissivity;
$\varkappa$,	absorption coefficient;
σ,	Boltzmann constant;
χ^*,	$k\varkappa\nabla^2 T^* - 3k\varkappa T^* - 4\sigma T^{*4}$;
χ,	$\chi^*/(4\sigma T_0^{*4})$;
ϕ,	$-(\chi + 3NT)$, radiation potential.

Subscripts

s,	value on boundary;
0,	value at $x, y = 0$;
l,	value at $x, y = l$.

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† The first author is now with the Department of Mechanical Engineering, University of Ottawa: Ottawa, Canada.

INTRODUCTION

THE PROBLEM of energy transfer in a conducting, absorbing and emitting medium has been considered highly non-linear [1], due to the presence of the re-emission term (T^4 for a gray medium). However, for a one dimensional problem it has been shown by Chang [2] that this non-linearity is not as severe as was thought and the re-emission term T^4 can be linearized by functions which can be chosen in a number of ways. In this note a two-dimensional problem, illustrated in Fig. 1, is analyzed according to the differential formulation. The basic non-linear equation is first solved numerically and this solution is then compared to the solution of the linearized equation. The following simplifying assumptions are used: (i) local thermodynamic equilibrium exists; (ii) the medium is gray, with a refractive index of one and constant properties; (iii) scattering is negligible; and (iv) the boundary surfaces are gray and emit and absorb radiation diffusely.

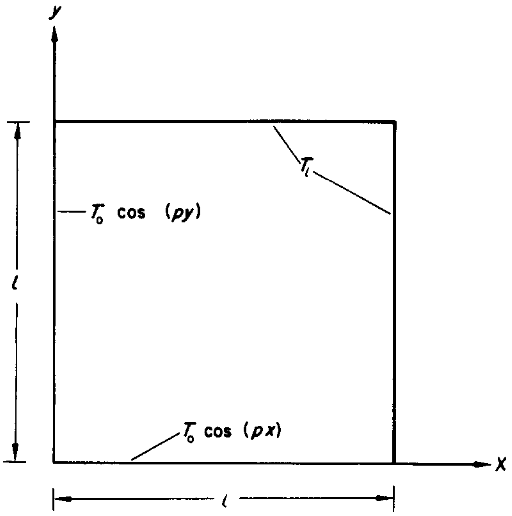

FIG. 1. Geometry of the problem.

BASIC EQUATIONS AND THEIR SOLUTIONS

With the simplifying assumptions and the dimensionless quantities defined above, the governing equations, which have been discussed elsewhere [2, 3], are as follows

$$N\nabla^2 T - 3NT - T^4 = \chi \quad (1)$$

$$\nabla^2 \chi = 0 \quad (2)$$

with the boundary conditions on χ and T

$$\left(\frac{\partial \chi}{\partial n}\right)_s = h_s(\chi_s - f_s) \quad (3)$$

$$f_s = -3NT_s - T_s^4 + \frac{3N}{h_s}\left(\frac{\partial T}{\partial n}\right)_s$$

$$T(0, y) = T_0 \cos py, \qquad T(x, 0) = T_0 \cos px \quad (4)$$

$$T(l, y) = T(x, l) = T_l.$$

Once χ is found the heat flux is given by

$$\vec{q} = \tfrac{4}{3}\nabla\chi. \quad (5)$$

The formal solution of (2) for χ satisfying (3) is readily found as

$$\chi(x, y) = \frac{1}{4\pi}\int_{s'} f_s(s')\left(\frac{\partial G}{\partial n'}\right)_{s'} \mathrm{d}s' \quad (6)$$

where $G(x, y; x', y')$ is the Green's function associated with χ and can be easily found as [4]

$$G = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{4\pi}{\lambda_n^2 + \lambda_m^2}\,\Psi_m(x)\,\Psi_m(x')\,\Psi_n(y)\,\Psi_n(y')$$

where

$$\Psi_n(x) = \gamma_n(\lambda_n \cos \lambda_n x + h_s \sin \lambda_n x)$$

$$\gamma_n = \left[\frac{2}{l(\lambda_n^2 + h_s^2) + 2h_s}\right]^{\frac{1}{2}}$$

and λ_n are the roots of

$$\tan \lambda l = \frac{2h_s\lambda}{\lambda^2 - h_s^2}.$$

It can be shown that as $N \to \infty$, $\chi \to -3NT$, the pure conduction case; as $N \to 0$, $\chi \to -T^4$, the pure radiation case; and as $(\partial T/\partial n)_s$ and $(\partial \chi/\partial n)_s \to 0$, $\chi \to -3NT - T^4$, the Rosseland diffusion approximation. Solutions for these special cases will be used later.

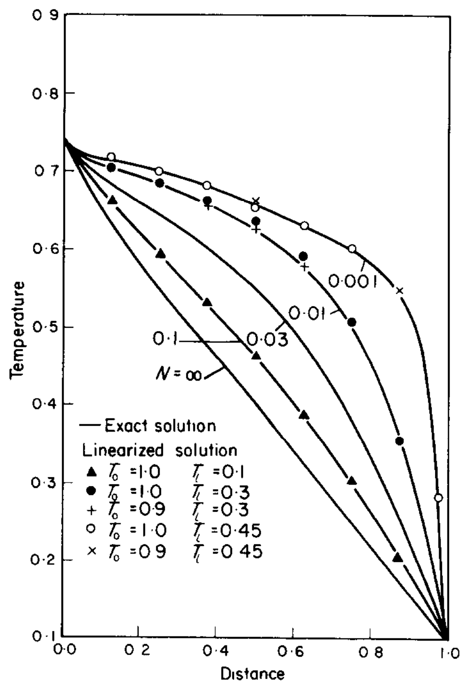

FIG. 2. Temperature distribution at $(x, l/2)$ or $(l/2, y)$.

Exact solution of (1). With $\chi(x, y)$ given by (6), equation (1) was solved numerically by a finite difference method. A graded mesh was used with a small length increment near the boundaries. The resultant set of non-linear, algebraic, equations was then solved by iteration with a convergence criterion of 0·1 per cent. Some of the calculated results for the temperature distribution at $(x, l/2)$ or $(l/2, y)$ and $(x = y)$ are shown as solid curves labeled exact in Figs. 2 and 3. The heat flux at the surface $(0, y)$ or $(x, 0)$ is given in Fig. 4. These calculations are for $T_0 = 1$, $T_l = 0{\cdot}1$, $l = 1$ and black surfaces, i.e. $h_s = 1{\cdot}5$.

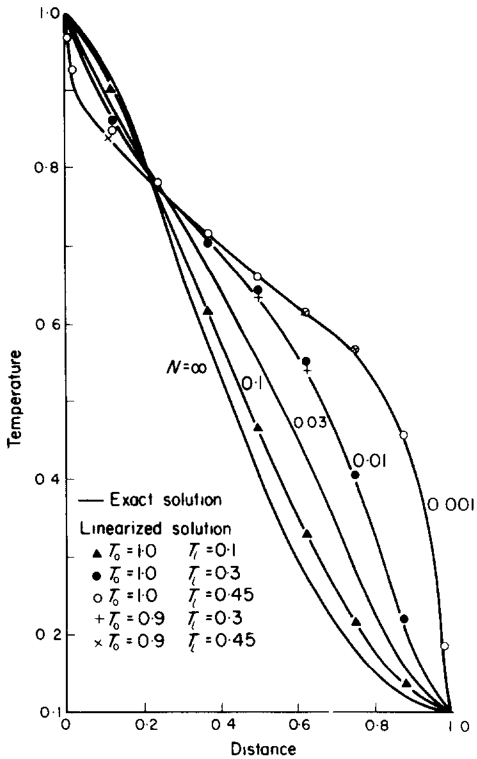

FIG. 3. Temperature distribution at $x = y$.

Approximate solution of (1). We linearize the re-emission term T^4 by writing $T^4 = 4T_a^3 T - 3T_a^4$ where $T_a(x, y)$ is a known function. Equation (1) is then

$$N\nabla^2 T - (3N + 4T_a^3)\, T = \chi - 3T_a^4 \qquad (7)$$

with the same boundary conditions as (4). The function $T_a(x, y)$ may be taken as that obtained from the Rosseland diffusion approximation, or that of pure conduction, i.e. $\chi(x, y, N \to \infty)$. The latter is simpler, but for small values of N, the following modification of the boundary condition is employed [2]:

$$\chi(0, y, N = 0) < T(0, y) \cos(py) < T_0 \cos(py)$$

$$T_l < T(l, y) < \chi(l, y, N = 0).$$

The same holds for $T(x, 0)$ and $T(x, l)$. Equation (7) was solved numerically by using T_a as the conduction solution.

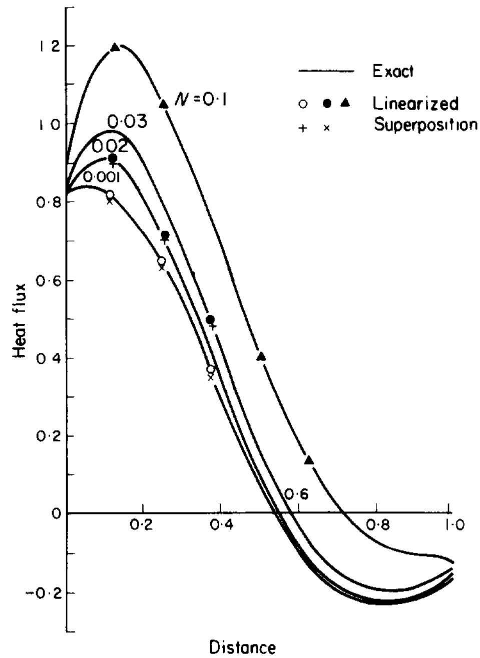

FIG. 4. Heat flux at $(0, y)$ or $(x, 0)$.

Some results are shown in Figs. 2 and 3. Maximum errors of the temperature and heat flux are shown in Table 1, for cases where the temperatures at the hotter surfaces were not modified.

Table 1. Differences between linearized and exact solutions

N	Max, % difference	
	Heat flux	Temperature
⩾1	0	0
0·1	0·065	1·50
0·03	0·141	2·47
0·01	0·022	3·02

Solution of (7) obtained by using T_a as that of the Rosseland diffusion approximation yielded virtually the same results. The error in the heat flux is negligibly small while that of the temperature is only a few per cent. When T_0 was modified as indicated in Figs. 2 and 3, the maximum error in temperature was reduced to less than 1 per cent. If only the heat flux is of interest, the superposition of conduction and radiation gives reasonably good results,

$$\vec{q}(x,0) = 3N\,\nabla\chi(x,0,N\to\infty) + \tfrac{4}{3}\nabla\chi(x,0,N\to 0). \quad (8)$$

DISCUSSION

It may be noted in Fig. 4 that in order to maintain the prescribed surface temperatures, heat is to be applied along a part of the surfaces at $x, y = 0$ and removed along the remainder. For $N > 1{\cdot}0$ (conduction predominating), heat flows into the medium along the entire hotter surfaces. The theoretical basis of the linearization procedure was discussed for one-dimensional problems in [3] where it was shown that the radiation-potential profile is not sensitive to the variation of N. For the present two-dimensional problem, calculated curves of $\chi + 3NT$, the radiation potential, exhibit the same character, i.e. their shapes do not change greatly with N, and consequently the success of the linearization is assured.

Further discussions pertaining to the effects of absorption and re-emission on the temperature field can be made by rewriting (1) in the form,

$$\nabla^2 T = -\frac{1}{N}(\phi - T^4) \quad (9)$$

where ϕ and T^4 represent, respectively, the radiant energy absorbed and re-emitted [3]. Obviously, the temperature will be higher than that of pure conduction for $\phi > T^4$ and lower for $\phi < T^4$. It is seen from Fig. 3 that re-emission predominates over absorption only in a small region near the hotter corner, and the latter is more important than the former in a large part of the medium. At smaller values of N, the difference between absorption and re-emission is magnified and hence the larger is the difference between the actual temperature and that of pure conduction. Presumably, the linearization procedure may apply as well when the convective process is involved and an approximate solution of (7) by variational method could be developed.

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DIFFUSE FREE CONVECTION IN A JET ABOVE AN AXIALLY SYMMETRIC ORIFICE DURING HYDROGEN OUTFLOW INTO AMBIENT AIR

K. BRODOWICZ
Technical University of Warsaw, Poland

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NOMENCLATURE

- x, vertical co-ordinate and orifice axis of symmetry;
- y, horizontal co-ordinate in the plane considered;
- z, horizontal co-ordinate;
- r, horizontal polar co-ordinate, $r^2 = y^2 + z^2$;
- S, displacement of interferometer fringes;
- n, refractive index of gas;
- β, concentration coefficient of volumetric expansion;
- C, volume concentration of hydrogen in air;
- D, diffusivity;
- V_{H_2}, volumetric flow rate of hydrogen;
- Gr, Grashof number for mass transfer;
- Sc, Schmidt number;
- Θ, concentration difference;
- h, dimensionless concentration function;
- ρ, density;
- g, gravitational acceleration;
- ν, kinematic viscosity.

INTRODUCTION

Diffusive free convection in a jet produced by hydrogen outflow into ambient air is characterized by velocity and concentration fields. Boundary conditions of the field depend on the outflow geometry.